TRANSACTIONS
OF THE
ILLUMINATING ENGINEERING SOCIETY

FOUNDED IN LONDON 1909

INCORPORATED 1930

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VOLUME XVI
No. 5 — 1951

32 VICTORIA ST
LONDON, SW1

Annual Subscription to Non-Members - £2.2.0
Price per Number to Non-Members - 5s. 0d.

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Long Linear Light Sources

An Analysis of the Infinite Linear Source and its Applications to Fluorescent Lamp Installations

BY H. D. EINHORN, Dr.Ing., Ph.D. *(Member)*

Summary

A new concept which should facilitate the calculation of illumination from long linear light sources is proposed and a general theory of the infinite linear source is given.

The errors incurred when applying infinite source relations to finite layouts are assessed for three types of sources.

How the results of this analysis can be applied to practical design tasks is shown in a few examples of fluorescent lamp layouts.

List of Symbols

F = luminous flux.
E = illumination.

The following quantities are defined for an infinite linear source (I.L.S.):

I = intensity per unit length of an I.L.S.
I_o = intensity per unit length of an I.L.S., normal to its axis (Fig. 2).
I_ψ = intensity per unit length of an I.L.S., at an angle ψ (Fig. 2).
I' = intensity of separate lamp (in a row, forming an I.L.S.: Sec. 3.3).
J = sector flux of I.L.S. (defined in Sec. 2).
J_o = sector flux of uniform I.L.S.
J_m = maximum sector flux of non-uniform I.L.S.

The following dimensions are shown in Figs. 2 to 8:—

R = radial distance from linear source.
L = length of finite linear source.
X = axial distance from end of finite source.
D = diagonal distance from end of finite source.
H = height of horizontal source above horizontal plane (Fig. 3).
C = centre distance of separate lamps (forming long linear source).
Z = sideways displacement of object point (Fig. 3).

The following angles are defined by Fig. 2:

ψ = axial angle, in plane of source.
θ = radial angle, in plane normal to source.
α = axial tilt of illuminated surface.
β = tilt of illuminated surface in plane normal to source.
ψ-*function* = variation of $I\psi$ with ψ
Δ = fractional reduction of illumination on a parallel plane near the end of a semi-infinite source.

Dr Einhorn is at the University of Cape Town, Union of South Africa Manuscript received May 26, 1950, and in revised form July 24, 1950.

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(1) Introduction

In many modern installations fluorescent lamps are arranged in long linear layouts, e.g., by mounting fluorescent fittings end to end or by using them in narrow ceiling panels, troughs or coves, for general lighting as well as for local lighting along work benches and desks. These types of layout lend themselves to very easy quantitative treatment, if they are considered as infinite linear sources. It is well known that in this case the illumination decreases inversely with the first power of the distance from the source, *not* with the inverse square of the distance, and the treatment of some special cases of infinite sources is generally known [1] [2]. Nevertheless, the number of illuminating engineers who make use of this very simple relation for estimates of illumination values is comparatively small.

While the Lumen-Method and more recently the Interreflection-Method for assessing average illumination values and brightness distribution obtained from general lighting are the main tools of the designer, point-by-point calculations can be very useful for the design of local lighting, for many special applications, for quick estimates of minimum values at particular points, for the study of evenness of illumination and in photometric tasks.

The infinite linear source relation in particular is so simple, that it would warrant much greater application. There are, however, three impediments to its use in practice:

(a) Absence of a method of dealing with *non*-uniform linear sources, e.g., a row of fluorescent lamps in industrial reflectors.
(b) Uncertainty about the property of sources which are not perfectly diffusing, such as a row of point sources, louvred lamps, a row of fluorescent lamps.
(c) Since no source is really infinite, uncertainty about the error involved in any practical case due to the finite length of the source.

This paper proposes to deal with these three difficulties by

(a) Suggesting the use of a new concept for dealing with non-uniform linear sources corresponding to the use of "intensity" for point sources.
(b) Making a general analysis of an infinite source without restriction to perfect diffusion or uniformity and proving two general theorems.

(c) Giving estimates of the error made or correction required when applying the infinite source relation to practical layouts containing finite linear sources.

Curves calculated for three special cases will be given in order to convey an idea of what to expect. A few examples of applications will demonstrate the simplicity of the method.

(2) Proposal of a New Quantity: "Sector Flux" (J)

(2.1) The Concept of J

Calculations for infinite linear sources can be based on the lamp flux per unit length if restricted to *uniform* sources. This would apply to bare fluorescent lamps or perhaps to lamps in cylindrical diffusing fittings. In the majority of cases we have, however, to deal with *non*-uniform sources, such as industrial reflectors or panels.

In this general case we could describe the light distribution by means of a polar curve in a plane normal to the direction of the linear source.

The question arises then: What is the dimension of a vector in this polar curve? It is not an intensity as in the case of a polar curve for a point source, but a quantity, which to the author's knowledge has not been used hitherto. It will be called "Sector Flux" in this paper and denoted by the letter J. It is defined (see Fig. 1), as "*Flux per unit length per unit angle.*"

$$J = \frac{dF}{dL\ d\theta} \qquad \ldots \qquad \ldots \qquad (2.1)$$

From this definition it follows that the correct unit for the sector flux would be: *Lumen per foot per radian;* one might shorten this to "lm./ft. r."

It is with great reluctance that the author proposes to add a new concept to the already heavily burdened illumination terminology. But new applications require new ideas and definitions.

The J-concept will facilitate the understanding of long sources which are used in practice to an increasing extent. Its application can be very wide: to tubular lamps with or without reflectors, to louvred fittings as well as rows of closely spaced point sources (see definition of line source in Sec. 3.1 below).

A further discussion of the J-concept is given in App. 1.

(2.2) Sector Flux and Lamp Flux

From the definition of J it follows immediately that *the lamp flux per unit length* of the installation is

$$F_1 = \int_0^{2\pi} J(\theta)d\theta \qquad \ldots \qquad \ldots \qquad (2.2)$$

In the case of a uniform linear source, such as a bare fluorescent lamp, where $J(\theta) = J_0 = \text{constant}$,

$$F_{1U} = 2\pi J_0 \qquad \ldots \qquad \ldots \qquad (2.3)$$

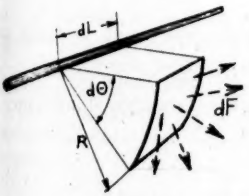


Fig. 1.

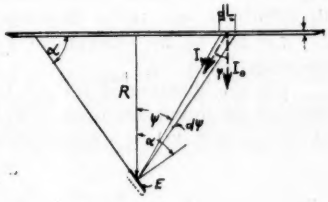


Fig. 2.

In the case of a plane diffusing source such as a long diffusing panel, where $J_\theta = J_m \cos\theta$, one can prove by a simple integration that

$$F_{1_p} = 2\,J_m \qquad \ldots \qquad \ldots \qquad (2.4)$$

In the general case of any linear source with a reflector, if the polar curve $J(\theta)$ is given, the flux per unit length is simply obtained by taking the average of all J-values measured at equal angular intervals and multiplying by 2π.

(Methods such as Rousseau Diagram, Russell angles, or Zone Factors which are used for evaluating polar curves representing *intensities* are *not* required here.)

(3) General Theory of the Infinite Linear Source

(3.1) Definition of Infinite Linear Source.

A general theory of the infinite linear source can now be developed and certain conclusions drawn from it. Some definitions will be given first.

(i) A source is linear if its thickness is negligible, i.e., very small compared with the object-distance ($t \ll R$), (see Fig. 2).

(ii) A linear source can be considered as infinite if its total axial length L is very large compared with the object-distance ($L \gg R$).

(iii) The radiating properties of the elements of the source should be constant along the length of the source, but a source can still be considered linear if it is "periodic," i.e., made up of elements which repeat themselves *ad infinitum*, provided the periodicity distance (C), or spacing of elements is smaller than the object-distance ($C < R$).

Most practical linear installations are periodic to a lesser or greater degree: for hot cathode fluorescent lamps mounted end to end, connectors and end darkening introduce a mild periodicity. More typical examples are: rows of point sources: fluorescent lamps with cross louvres. For discussion of periodicity see App. 5.

There shall be no restriction of the radial polar curve $J = f(\theta)$ (see Figs. 1, 2). In practice it is mainly determined by the type of reflector used. In the special case of a radially uniform source such as a bare fluorescent tube it is a circle with the vector origin in the centre.

There shall be no restriction as to axial distribution, i.e., the axial polar curve $I_\psi = f(\psi)$ of an element is unrestricted. (See Fig. 2). It is mainly determined by the character of the source (e.g. whether it is a row of point sources or a fluorescent lamp), and by certain features of the fitting such as cross louvring. In the well known special case of a perfectly diffusing source it is a circle with the vector origin on the circumference.

(3.2) General Equation

Let us consider the light distribution in any one plane for which $\Theta = \text{const.}$ (see Fig. 2), i.e. in a plane which contains the source itself as a boundary line and extends to infinity in one radial direction.

Let $I\psi$ be the intensity of an element of the source of unit length (1 foot) in the direction ψ;

Let the illuminated surface be placed at a distance R (feet) from the nearest point of the source and tilted in an axial direction by an angle α; then the illumination at this surface due to a source element of length dL where

$$dL = R\,sec^2\psi\,d\psi \text{ is}$$

$$dE = \frac{I_\psi \quad dL}{R^2}\cos^2\psi\,\cos\,(\alpha - \psi) = \frac{I_\psi}{R}\,\cos\,(\alpha - \psi)\,d\psi$$

and the illumination due to the complete source is

$$E = \frac{1}{R} \int_{\psi=\alpha-\pi/2}^{\psi=\pi/2} I_\psi \cos(\alpha - \psi)\, d\psi \quad \ldots \quad \ldots \quad (3.1)$$

The fact that R appears only outside the integral permits the statement:

THEOREM I.—*The inverse first-power law applies to infinite line sources universally, i.e., for any emission characteristic* [*of* ψ*-function:* $I_\psi = f(\psi)$], *and for any orientation* (α) *of the illuminated surface.*

A tilt in a plane at right angles to the source by an angle β (see Fig. 2) introduces a factor cos β which is of no particular interest in this analysis since it is merely a constant in the integration.

(3.3) Sector-Flux J and Ψ-Function

The sector-flux (or J value) is fully determined by the ψ-function or axial polar curve of an element of the source.

The relation is quite simple (for proof see App. 3).

$$J = \int_{\psi=-\pi/2}^{\psi=+\pi/2} I_\psi \cos\psi\, d\psi \quad \ldots \quad \ldots \quad (3.2$$

Both J and I_ψ are defined and used here as per running foot of a long source. In practice, a long line source is usually made up of separate lamps and the polar curve $I'_\psi = f(\psi)$ of each of those lamps may be given or measured.

If the lamps are so spaced that the distance between their centres is C feet,

$$I_\psi = I'_\psi / C \text{ and}$$

$$J = \frac{1}{C} \int I_\psi \cos\psi\, d\psi \quad \ldots \quad \ldots \quad (3.3)$$

(3.4) Illumination on Surfaces Parallel to the Source ($\alpha = 0$).

From equation (3. 1) and (3. 3) follows immediately a very useful relation for the important special case where the illuminated surface is parallel to the source, i.e., $\alpha=0$:

$$E = \frac{J}{R} \quad \ldots \quad \ldots \quad (3.4)$$

Expressed in words, it is

THEOREM II.—*The illumination (in lumens per sq. ft.) on a surface parallel to an infinite linear source is equal to the sector flux (in lumens per ft. per radian) divided by the distance R (in ft.) irrespective of the emission characteristic (ψ-function) of the source.*

This relation is both the reason for suggesting J as a useful concept and the basis for measuring it in a simple way as $E \times R$.

If the object surface is parallel to the source ($\alpha = O$), but its normal does not intersect the source ($\beta \neq O$), as in the common practical case of a horizontal lamp above a horizontal working area where the point of interest P does not lie directly below the lamp (see Fig. 3 inset), the equation for the illumination at P is still very simple:

$$E = \frac{J}{R} \cos\beta = \frac{J\,H}{H^2 + Z^2} \quad \ldots \quad \ldots \quad (3.5)$$

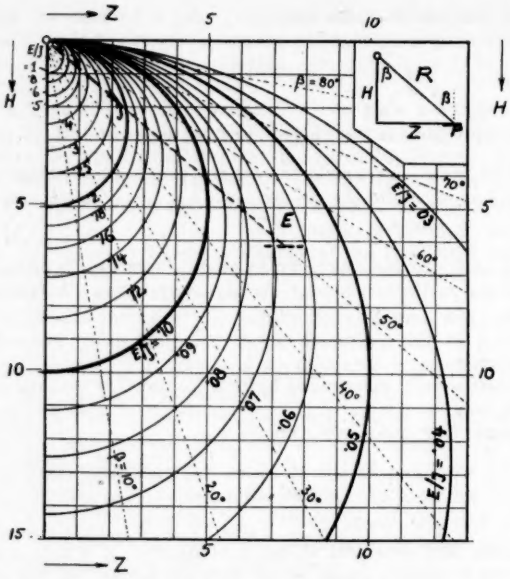


Fig. 3.

J is of course the sector flux in the direction of P. This relation is more convenient to use than the corresponding one for point sources, which contains an exponent of 3/2 in the denominator.

The locus-diagram (Fig. 3) is based on equation (3.5). The curves represent positions of constant E/J in a plane normal to the source. One can prove that they are circles of diameter J/E. Note that E is the illumination on horizontal surfaces, *not* on the plane of the diagram.

Fig. 3 can be applied to non-uniform sources, where $J(\theta) \neq$ const., e.g., in the case of fluorescent lamps in industrial reflectors, because the *ratios* E/J are valid for any value of J in the direction of P.

An example will show how simple its use is. Let the sector flux of a row of fluorescent fittings in a direction of 50 deg. be 90 lm./ft.r. (see arrow in heavy broken lines in Fig. 3). At a distance of 9.6 ft. in this direction E/J can be read off as .067. Hence the illumination there (due to direct light from this row of lamps) is .067 × 90 = 6.0 lm./ft.^{2} on a horizontal plane.

(3.5) Illumination on Inclined Surfaces ($\alpha \neq 0$)

If the surface illuminated is not parallel to the infinite source but inclined to it, the illumination is not fully determined by sector flux and distance, but depends on the ψ-function, i.e., the radiation properties of the source as well.

Let us consider a few special cases for which evaluation is relatively simple. Most practical cases are similar to one of them or a mixture of two of them.

(i) "*Constant Intensity*" line source, such as a row of uniform point sources or certain discharge lamps, for which the *intensity of an element* is the same in all directions:

$$I_\psi = I_0 \quad \ldots \quad \ldots \quad \ldots \quad (3.6)$$

(ii) A *perfectly diffusing* line source, of which the *luminance* is the same in all directions, resulting in a cosine law for the intensities:

$$I_\psi = I_\circ \cos\psi \quad \ldots \quad \ldots \quad \ldots \quad (3.7)$$

(iii) A *cosine-squared radiating* source, of which the luminance falls off according to a cosine law, the intensity according to a cosine-squared law :

$$I_\psi = I_\circ \cos^2\psi \quad \ldots \quad \ldots \quad \ldots \quad (3.8)$$

Table I, column 2, shows the relation between sector flux and intensity per unit length for these three cases.

Table 1.

Infinite Source.

	1	2	3
Case	Type of Source (ψ-function)	J—$I_\circ$ relation	Illumination (for any orientation of object surface)
(i)	$I_\psi = I_\circ$	$J = 2\,I_\circ$	$E = \frac{1}{2}\frac{J}{R}(1+\cos\alpha)\cos\beta$
(ii)	$I_\psi = I_\circ \cos\psi$	$J = \frac{\pi}{2} I_\circ$	$E = \frac{1}{\pi}\frac{J}{R}[\sin\alpha + (\pi - \alpha)\cos\alpha]\cos\beta$
(iii)	$I_\psi = I_\circ \cos^2\psi$	$J = \frac{4}{3} I_\circ$	$E = \frac{1}{4}\frac{J}{R}(1+\cos\alpha)^2\cos\beta$

The type of source most commonly treated in the literature is the diffusing one (case ii). For a discussion of the constant intensity source (case i) see Reference 1, page 228.

For fluorescent lamp practice, cases (ii) and (iii) are of importance. (See Sec. 5.)

Introducing the ψ-functions (equ. 3.6, 3.7, 3.8) into equation (3.1) the results shown in Table I, column 3, and Fig. 4 are obtained. (See App. 4).

As one should expect from theorem II, the relation for the illumination becomes $E = J/R$ for $\alpha = 0$, in all three cases. For other angles the illumination is less, as shown in the figure. This is, however, by no means a universal rule. One could design sources where the illumination *increases* for tilted object surfaces. On the other hand by means of deep louvres or focused sources a falling off proportional to cos α could be obtained.

Curves like those shown in Fig. 4 indicate the relation between the illumination of horizontal and vertical or tilted surfaces and may sometimes give an idea of the character of a layout with respect to contrast, shadows and plasticity, although the difference between the three cases treated above is probably not great enough to be of practical significance.

(4) Estimate of Corrections for Finite Source

The relations given for an infinite source, particularly the simple $E = J/R$ for parallel surfaces can be used as a practical approximation for a long *finite* linear source, provided the radial distance from the source is kept within certain limits, determined by the required accuracy of the estimate. The errors incurred when treating a finite source as infinite will now be discussed. The result can be used either as a limit of the straight forward application of the infinite-source relation or for correcting the result if it is desired to extend this limit.

(4.1) Illumination on Parallel Surface near the end of a Semi-Infinite Source

Consider a horizontal line source which extends to infinity (or is very long) in *one* direction (See Fig. 5): How will the illumination vary on a horizontal plane, along a line straight below the source?

One general statement can be made immediately: Provided the ψ-function is symmetrical, i.e., $I(\psi)=I(-\psi)$, the illumination below the end of the source (X=0) is one half that below a point of the source very far from the end. This applies to the special cases under consideration and indeed to most practical cases.

This statement based on the argument that one half of the infinite source (viz., that for $X<0$) has been taken away must apply to any semi-infinite source, irrespective of its radiation characteristics expressed in the ψ-function.

The actual rate of falling off the illumination does, however, depend on this ψ-function.

For the three special cases treated in Section 3.5 (i) constant intensity-type (ii) perfectly diffusing (iii) cosine-squared radiator, the result is as given in Table II, column 1, and Fig 5. The derivation is given in App 4.

It can be seen that the illumination falls off sharpest for the cosine^2 radiator, more gradually for a perfectly diffusing source, most gradually for a row of point sources.

In an ideally louvred fitting, i.e., with very long black cross louvres, closely spaced,

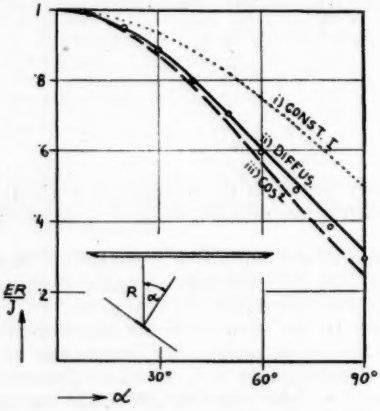
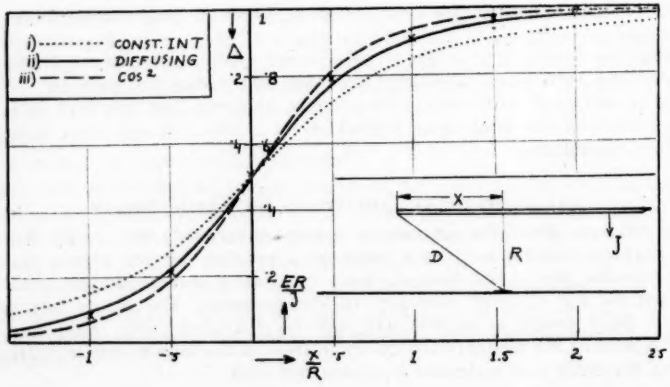


Fig. 4. *(Left)*

Fig. 5. *(Below)*



the illumination for $X<O$ will be zero, for $X>O$ it will be constant and equal to J/R, with an abrupt drop at $X=O$. In practice, for lamps fitted with matt, light coloured louvres, the cosine squared law would probably give a fair approximation,

In most fluorescent lamp installations whether louvred or not, the falling-off ($\triangle$) near the end is less than 10 per cent. for $R<X$, i.e., if the radial (vertical) distance of the object surface does not exceed the axial (horizontal) distance from the end of the (horizontal) lamp.

(4.2) Illumination on Parallel Surface Below Centre of Long Source

To give an estimate of the accuracy with which the simple relation $E=J/R$ can be applied to long linear sources illuminating a surface parallel to the lamp, the precise relations for the illumination below the centre of a finite lamp are given in Table II, column 2, and Fig. 6, again for the three special cases treated previously.

The equations and curves show that while for a "constant intensity" source such as a row of point sources the distance from the lamp should not exceed $\frac{1}{4}$-lamp length if the J/R relation is to hold with an accuracy of 10 per cent., for fluorescent lamps one may go as far as 0.4 lamp length, for louvred lamps probably even to $\frac{1}{2}$-lamp length for the same accuracy.

Table 2.

Semi-Infinite and Finite Source.

	1	2	3
Case	Illum. from Semi-Infin. Source For $\alpha = 0$; $\beta = 0$	Illum. from Finite Source	
		Below centre for $\alpha = 0$; $\beta = 0$	For $\alpha = 90°$; $\beta = 0$
(i)	$E = \frac{J}{R}(\frac{1}{2} + \frac{1}{2}\frac{X}{D})$	$E = \frac{J}{R}\frac{X}{D}$	$E = \frac{1}{2}\frac{J}{R}(1 - \frac{R}{D})$
(ii)	$E = \frac{J}{R}(\frac{1}{2} + \frac{1}{\pi}\tan^{-1}\frac{X}{R} + \frac{1}{\pi}\frac{XR}{D^2})$	$E = \frac{J}{R}(\frac{2}{\pi}\tan^{-1}\frac{X}{R} + \frac{2}{\pi}\frac{XR}{D^2})$	$E = \frac{1}{\pi}\frac{J}{R}\frac{X^2}{D^2}$
(iii)	$E = \frac{J}{R}(\frac{1}{2} + \frac{3}{4}\frac{X}{D} - \frac{1}{4}\frac{X^3}{D^3})$	$E = \frac{J}{R}(\frac{3}{2}\frac{X}{D} - \frac{1}{2}\frac{X^3}{D^3})$	$E = \frac{1}{4}\frac{J}{R}(1 - \frac{R^3}{D^3})$

R = Radial object distance from lamp.

X = Axial component of object distance from end of lamp.

$D = \sqrt{R^2 + X^2}$ = actual distance of object from end of lamp.

(4.3) Illumination on a Parallel Surface Anywhere Below Finite Source

The graphs Fig. 5 or 6 can be used to obtain the illumination anywhere below a finite linear source.

Considering Fig. 7 it is easily seen that at point P

$$E = \frac{J}{R}(1 - \triangle_a - \triangle_b) \ldots \qquad \ldots \qquad (4.1)$$

where $\triangle$ is the fractional difference between the illumination near the end of a semi-infinite source and that below the infinite source. $\triangle_a$ and $\triangle_b$ can be taken directly from Fig. 5 or 6 for the ratios X_a/R and X_b/R.

Below the centre of a finite lamp the correction necessary is therefore equal to $2\triangle$ taken for X/R from Fig. 5, where X is half the lamp length.

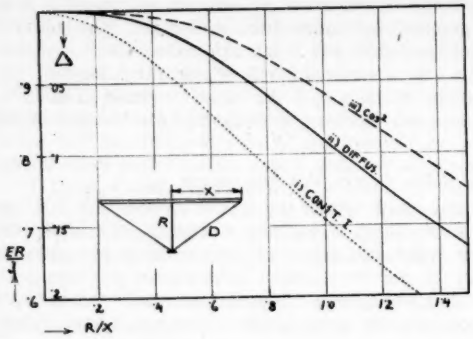


Fig. 6.

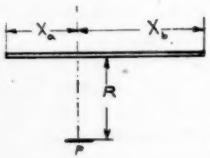


Fig. 7.

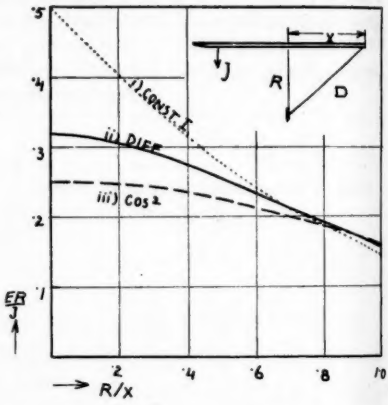


Fig. 8.

(4.4) Illumination on Surface at Right Angles to Lamp

For the illumination on surfaces at right angles to the lamp itself, e.g., on a vertical surface in case of a horizontal lamp the effect of the finite lamp length is rather greater than for parallel surfaces.

Table II, column 3 and Fig. 8 show the relation for the three special cases considered previously. (For proof see App. 4.)

(5) Application to Fluorescent Lamp Installations

(5.1) J—I_o Relation

As mentioned in Sec. 3.5 cases (ii) and (iii) are applicable to fluorescent lamps.

Most unlouvred fluorescent lamps can be represented as a mixture of cosine and cosine-squared radiators([3]). Bare lamps are about midway between case (ii) and case (iii) tending slightly towards case (ii). The same applies to lamps with specular reflectors. Lamps with diffusing reflectors will tend in their behaviour still more towards case (ii), while cross-louvred lamps of the more usual designs can be represented as cosine-squared radiating sources (case iii) with an accuracy adequate for most practical purposes.

From this follows that in the absence of further data available, the following

relations between the sector fluxes of a fluorescent lamp fitting and its intensities in a plane normal to the lamp axis might be expected:

Bare Lamp	$J/I_\circ = 1.47$
Lamp and Diffusing Reflector	$J/I_\circ = 1.50$
Louvred Lamp	$J/I_\circ = 1.2 - 1.4$

.. (5.1)

$I_\circ$ is the intensity *per foot of installation*. For example, if 4-ft. lamps are mounted at 5-ft. centres, i.e., with gaps of 1 ft. between ends of lamps, then $I_\circ = \frac{I'_\circ}{5}$, where $I'_\circ$ is the normal intensity of one fitting.

(5.2) Approximate Relations

For practical fluorescent lamps with or without reflectors, but without cross-louvres, the following approximation might be used:

$$\triangle = .075 \left(\frac{R}{X}\right)^2 \quad \ldots \quad \ldots \quad (5.2)$$

This deviates by less than $\pm$ 0.01 from the mean curve between ideal cases (ii) and (iii) over a range: $0 < R < 1.2\ L$.

Below the centre of a lamp with reflector, the illumination on a nearby parallel object surface is therefore approximately:

$$E \doteqdot \frac{J}{R}\left(1 - .15\,\frac{R^2}{X^2}\right) \doteqdot \frac{1.5\ I'_\circ}{R\ L}\left(1 - .6\,\frac{R^2}{L^2}\right), \ \ldots \quad (5.3)$$

where L the lamp length.

(6) Experimental Checks

(6.1) Laboratory Measurements

The relations given were checked by measurements on eight bare 3-ft. fluorescent lamps mounted end-to-end in a laboratory with black walls and ceiling, giving 2 per cent. stray light, for which a correction was made.

A barrier-layer cell was used for the measurements with a specially shaped cover of diffusing acrylic sheet and a shadow screen to give cosine-response with an accuracy of 2 per cent. up to an angle of 75 deg. and an error below 3 per cent. of the cosine zero value between 75 deg. and 90 deg.

Some experimental points are shown in Figs. 4 and 5. It can be seen that agreement with the calculated curves is certainly good enough for practical purposes.

(6.2) Practical Measurements

In order to test the application to practical cases where reflected light adds appreciably to the illumination, some rough measurements were made in a section of the university library, where parallel book shelves 4 ft. 4 in. apart were lit by 4-ft. fluorescent lamps, spaced 7ft. 9in. (centre to centre), mounted without reflectors against a white ceiling.

The illumination values measured varied between about 2 to 10 lm/sq. ft. Their *relative* values agreed with calculated ones to within about 20 per cent.; this discrepancy is due to different contributions from inter-reflected light and does not seem excessive considering the wide range of values assessed, and considering that the layout was not particularly suitable for any method of calculation.

(7) Practical Examples

(7.1) Bench Lighting

When laying out local lighting it is sometimes of interest to know the minimum illumination and the evenness of lighting level over the task area.

Take the following case (Fig. 9). A long work bench, 5ft. wide, is lit by twin

40-watt daylight fluorescent lamps in enamelled industrial reflectors mounted end to end (4ft. 3in. distance between centres), suspended 3ft. 6in. above the centre line of the bench.

The polar curve of the fitting shows that its intensity in a vertical direction is 570 candelas, in a direction of 35 deg. (towards the edge of the bench) 540 candelas.

The calculation is simple and given in the following table:

	Vertically below lamp	At edge of bench
Sector Flux (See Sec. 5.1)	$\frac{570\ (\text{cd})}{4.25\ (\text{ft.})}\ 1.5 = 200$ lm/ft.r.	$\frac{540}{4.25}1.5 = 190$ lm./ft.r.
Illum. (See equ. 3.6 or Fig. 3)	$\frac{200}{3.5} = 57$ lm./ft.^{2}	$\frac{190 \times 3.5}{3.5^2 + 2.5^2} = 36$ lm./ft.^{2}

Ratio of maximum/minimum illum. = 57/36 = 1.6.

If we wish to estimate the illumination at the edge of the bench, 2 ft. from the end of the row of lamps, we can see from Fig. 5 that for $\frac{X}{R} = \frac{2}{4.3} = .47$, the illumination is just over three-quarters of the value calculated above, i.e. it would be about 27 lm./ft.^{2}

(7.2) Optimum Position of Linear Source

We could have asked in the previous example: What is the optimum suspension height (X) which will give maximum illumination at the edge of a bench, 2 A (feet) wide, assuming constant J over the critical angular range from say 30 deg. to 60 deg? A similar question arises in lighting vertical surfaces such as pictures or blackboards: how far from the wall should a source with limited sector flux be placed to give maximum illumination at the lowest point of interest (A feet below the source)? From equation 3.5 : $E = \frac{J\,X}{X^2 + A^2}$; this is a maximum when $\frac{dE}{dX} = 0$, namely for $X = A$.

Hence one could formulate the rule: Place a *linear* source at such a distance from the plane lit as to obtain 45 deg. incidence of light at the furthest point of interest.

Like all rules of this type it should be used with caution: e.g., if evenness of

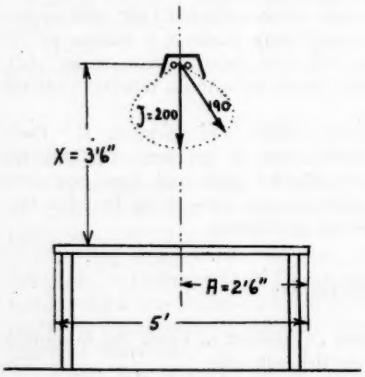


Fig. 9.

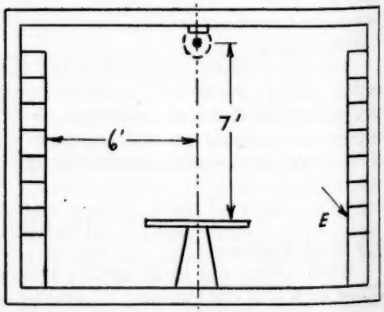


Fig. 10.

illumination matters more than minimum value, the source should be placed further away.

NOTE 1.—The assumption was made that we deal either with a uniform source over the critical angular range or that a maximum J is given with the possibility of tilting the source.

NOTE 2.—The optimum position for a *point* source would be different: $X' = A/\sqrt{2}$.

(7.3) Filing Room or Library

Sometimes vertical as well as horizontal surfaces are of interest. In this case the average horizontal illumination obtained from the conventional lumen-method has little meaning and a rough estimate of the illumination at some key points would convey better information. Consider a filing room or library stackroom 22 ft. x 12 ft. (between shelves) as shown in Fig. 10, illuminated by a row of four single 5-ft. 80-watt fluorescent lamps mounted end to end at 5 ft. 6 in. centre distance along the centre of the ceiling.

Assuming the lamps are in uniformly diffusing fittings and neglecting light reflected from the ceiling, let us estimate the following illumination values, all at points 7 ft. below the lamp:

(a) on a horizontal table in the centre of the room,
(b) on vertical shelves, centre of long wall,
(c) on vertical shelves, end of long wall,
(d) on vertical shelves, centre of short wall,
(e) on vertical shelves, end of short wall.

From equation 2.3 and 3.3 $J = \dfrac{F}{2\pi C} = \dfrac{3040}{2\pi 5.5} = 88$ lm/ft.r.

Taking the source as infinite for a, b, and semi-infinite for c, d, e the following results are obtained:

Case	Position	Reference	Calculation	Illumin. (lm./ft.^{2})
a	Table	Eq. (3.4)	$88/7 =$	12.6
b	Centre long wall	Eq. (3.5) or Fig. 3	For $H = 6$, $Z = 7$ (vertical surface.) $E = .071\ J$	6.2
c	End long wall	—	Half previous value	3.1
d	Centre short wall	Table I ; Fig. 4	$E = \left(\frac{1}{4} \text{ to } \frac{1}{\pi}\right) J/R$, say $.29\ J/R$	3.6
e	End short wall	Table I ; Fig. 4	$E = \dfrac{.29 \times 88}{\sqrt{36 + 49}}$	2.8

The errors made in neglecting the finite length are estimated in the following table:—

Case	Reference	For X/R	$\triangle$	Error
a	Fig. 5 or 6	$10.5/7 = 1.5$	.03	6%
b	Fig. 5	$10.5/9.2 = 1.1$	.06	12%
c	Fig. 5	$21/9.2 = 2.3$	.03	3%
d	Fig. 8	$21/7 = 3$	—	7%
e	Fig. 8	$21/9.2 = 2.3$	—	12%

Although reflected light would add appreciably to the values calculated, some conclusions may be drawn from them (see Sec. 6.2):—

(i) The illumination of the short walls, particularly in the corners, is inferior.

The light throw of linear lamps is essentially at right angles to the axis (see [3] Sec. 3.2.1). Doubling of the lamps in the end fittings might be considered.

(ii) The average horizontal illumination as estimated from the Lumen Method would be about 13 lm./ft.^{2} for a utilisation factor of .35 and maintenance factor .8. As expected, a meaningless if not misleading figure.

(iii) The errors made when using the infinite source relations are 12 per cent. or less in this case: tolerable for most estimates.

(8) Conclusions

(a) To facilitate the calculation of illumination from linear arrangements of lamps, a new concept has been proposed in this paper: the "Sector Flux," defined as flux per unit length per unit angle.
The sector flux can be taken as the vector in polar curves for long linear sources.

(b) The illumination on a surface parallel to a long linear source is simply obtained as $\frac{\text{sector flux}}{\text{distance}} \times \cos\beta$ (where β is the angle of incidence, see Fig. 2).

(c) The illumination on surfaces not parallel with the source depends on the radiation characteristics of the latter. The inverse first power law is still valid. Three typical cases are treated in Sec. 3.5.

(d) For practical linear layouts which, of course, have a finite length, the infinite source relations form often fair approximations. The errors involved have been investigated in Sec. 4.

(e) The application to fluorescent lamp layouts, particularly the assessment of sector flux values from intensity data, is discussed in Sec. 5.

(f) Practical examples worked out showed the simplicity of using the infinite source relations given. The errors made in using them were assessed and seemed to be tolerable for practical purposes in the cases treated.

(g) The author hopes that some illuminating engineers, when dealing with linear layouts, will try out the new concept proposed and the relations given here. Only this practical test can determine their usefulness.

(9) References

(1) Higbie, H. H., Lighting Calculations. (Wiley, New York, 1934.)
(2) Moon, P., Scientific Basis of Illuminating Engineering. (McGraw-Hill, New York, 1936.)
(3) Einhorn, H. D., and Sauermann, J. D., Journ. Inst. Elect. Eng. **94, Part II,** 535 (1947).
(4) Moon, P., and Spencer, D. E., J. Opt. Soc. Amer, **36,** 666 (1946), and Illum. Eng. (U.S.A.) **42,** 614 (1947.)

Appendix I

Discussion of the Vector Flux-Concept

Analogy with Intensity

The vector flux concept for line sources has much in common with the intensity concept for point sources; but differs from it in other respects fundamentally.

In each case we deal with a specific angular flux: intensity—per steradian; sector flux—per radian.

In each case the illumination at a certain distance is determined by a simple relation, the inverse square law and the inverse first power law respectively.

In each case vectorial representation in a polar diagram is possible and informative.

The main difference, in the author's opinion, is that the intensity of a point source contains more complete information than the vector flux.

The vector flux concept gives no information on the axial light distribution of

the line-source (ψ-function). The illumination E on a parallel surface at distance R, corresponding to a vector flux RE, may be due to predominantly steep rays coming from a fairly narrow length of the lamp as in a louvred lamp. It may also be produced largely by rays impinging at a flat angle from far points of the source, as, e.g., in the extreme case of a row of sources with polar curves typical for street lighting. For this reason tilting the illuminated surface in an axial direction (variation of angle α) may have a quite unpredictable effect unless the ψ-function is known. There is no uncertainty of this type about a single point source, and tilting the illuminated surface has always a predictable effect, namely, variation of illumination according to a cosine law.

On the Utility of Intensity and Sector Flux

It is generally accepted to-day that "Flux" is the fundamental illumination-concept, not "Intensity," as is often found in older treatises on the subject.

A few years ago (4) the suggestion was even made to abolish "Intensity" as a standard concept.

This, in the author's opinion, goes too far. The study of conical bundles of rays which leads to the intensity concept is useful for investigating the directional properties of light sources; although theoretically restricted to point sources, it can always be applied to extended sources by considering initially surface elements.

Similarly, the concept of "Sector Flux" should be a useful special tool when dealing with line sources, and the limitation of its applicability should not detract from its utility within those limits.

Terminology

As an alternative name for "Sector Flux" the author had considered the term "Linear Intensity" or "Line-Intensity," to indicate the similarity mentioned above. Disadvantages of these terms are their length and the possibility of confusion.

Other names, however, might be considered. The author feels that the C.I.E. should decide on the best term for J, if it proves useful enough to warrant recognition as a standard term by the C.I.E.

The choice of the symbol J itself is subject to discussion, since it is an American standard symbol for radiant intensity. Confusion is, however, not likely to arise on account of the different field of application.

Appendix 2
Dimensions in Metric System

Sector Flux in the metric system would be measured in *lumens per metre per radian* (*lm./m.r.*), and could be called "Pharosent per radian" (4).

The intensity per unit length of source has to be taken then in *candelas per metre* (Sec. 3.2) and all dimensions should be in *metres* to obtain illumination values in *lux*.

Fig. 3 can be interpreted in metric units by taking the co-ordinates in metres, the sector flux in lm./m.r., to obtain the result in lux.

Appendix 3
Proof of Eq. (3.2)

Consider the flux (F'') due to unit length of the source through a thin strip (of width w) of a cylinder surface, the cylinder being concentric to the source and of radius r.

$$\textit{Then}: F'' = \int_{\xi=-\infty}^{\xi=+\infty} E\, w\, d\,\xi$$

where ξ is measured along the strip and E is the illumination along the strip.

In this equation $w = r\theta$

$$J = F''/\theta \text{ by definition}$$

$$E = \frac{I_\psi}{r^2} \cos^3\psi ;$$

$$d\xi = \frac{r\ d\psi}{\cos^2\psi}$$

Hence: $J = \frac{1}{\theta}\int \frac{I_\psi}{r^2}\cos^3\psi .\ r\theta .\ \frac{r d\psi}{\cos^2\psi}$ from $\psi = \pi/2$ to $-\pi/2$

$$= \int_{-\pi/2}^{+\pi/2} I_\psi \cos\psi\, d\psi$$

Appendix 4

Proof of Expressions in Tables I and II.

J — I° relations:

(*Table* I, *column* 2).

From equation 3.2 *follows*

Case (*i*) $\int_{-\pi/2}^{+\pi/2} \cos\psi\, d\psi = 2 \times \sin \pi/2 = 2$

Case (*ii*) $\int_{-\pi/2}^{+\pi/2} \cos^2\psi\, d\psi = \tfrac{1}{2} \sin\pi + \tfrac{1}{2}\pi = \frac{\pi}{2}$

Case (*iii*) $\int_{-\pi/2}^{+\pi/2} \cos^3\psi\, d\psi = 2\left(\sin \pi/2 - \frac{1}{3}\sin^3\frac{\pi}{2}\right) = \frac{4}{3}$

Illumination for $\alpha \neq 0$

(*Table* I, *column* 3).

From equation 3.1 *follows:*

Case (*i*) $E = \frac{I_0}{R}\int_{\psi = \alpha - \pi/2}^{\psi = \pi/2} \cos(\alpha - \psi)\, d\psi = \frac{I_\circ}{R}\Big|\sin(\psi - \alpha)\Big|_{\alpha-\pi/2}^{\pi/2} = \frac{I_\circ}{R}(1 + \cos\alpha)$ $\alpha - \pi/2$

Case (*ii*) $E = \frac{I_\circ}{R}\int_{\alpha-\pi/2}^{\pi/2} \cos\psi \cos(\alpha - \psi)\, d\psi$

$$= \frac{I_\circ}{R}\int_{\alpha-\pi/2}^{\pi/2} (\cos^2\psi \cos\alpha + \sin\psi\cos\psi\sin\alpha)\, d\psi = \frac{I_\circ}{2R}\left[\sin\alpha + (\pi - \alpha)\cos\alpha\right]$$

$$\textit{Case (iii)}\quad E = \frac{I_\circ}{R}\int_{a-\pi/2}^{\pi/2} \cos^2\psi \cos(a-\psi)\, d\psi$$

$$= \frac{I_\circ}{R}\int_{a-\pi/2}^{\pi/2} (\cos^3\psi \cos a + \cos^2\psi \sin\psi \sin a)\, d\psi$$

$$= \frac{Io}{R}\left| (\sin\psi - \frac{1}{3}\sin^3\psi)\cos a - \frac{1}{3}\cos^3\psi \sin a \right|_{a-\pi/2}^{\pi/2}$$

$$= \frac{I_\circ}{R}\left[\frac{1}{3} + \frac{2}{3}\cos a + \frac{1}{3}\cos^2 a\right] = \frac{I_\circ}{R}\frac{(1+\cos a)^2}{3}$$

Illumination from Semi-infinite source

($a=0$) (*Table II, column* 1) (*see fig.* 5).

From equation 3.1, *modifying limits* (*See fig.* 5, *inset*):

$$\textit{Case (i)}\quad E = \int_{\psi=-\sin^{-1}(X/D)}^{\psi=\pi/2} \frac{I_\circ}{R}\cos\psi\, d\psi = \frac{I_\circ}{R}\left|\sin\psi\right| = \frac{I_\circ}{R}\left(1+\frac{X}{D}\right)$$

$$= \frac{J}{2R}\left(1+\frac{X}{D}\right)$$

$$\textit{Case (ii)}\quad E = \int_{-\sin^{-1}(X/D)}^{\pi/2} \frac{I_\circ}{R}\cos^2\psi\, d\psi = \frac{I_\circ}{R}\left|\tfrac{1}{2}\psi + \tfrac{1}{2}\sin\psi\cos\psi\right|$$

$$= \frac{I_\circ}{R}\left(\frac{\pi}{4} + \tfrac{1}{2}\tan^{-1}\frac{X}{R} + \tfrac{1}{2}\frac{XR}{D^2}\right)$$

$$= \frac{J}{R}\left(\tfrac{1}{2} + \frac{1}{\pi}\tan^{-1}\frac{X}{R} + \frac{1}{\pi}\frac{XR}{D^2}\right)$$

$$\textit{Case (iii)}\quad E = \int_{-\sin^{-1}(X/D)}^{\pi/2} \frac{I_\circ}{R}\cos^3\psi\, d\psi = \frac{I_\circ}{R}\int_{n=-X/D}^{n=1}(1-n^2)\, dn,$$

where $n = \sin\psi$;

$$E = \frac{I_\circ}{R}\left|n - \frac{1}{3}n^3\right|_{-X/D}^{1} = \frac{I_\circ}{R}\left(\frac{2}{3} + \frac{X}{D} - \frac{1}{3}\frac{X^3}{D^3}\right)$$

$$= \frac{J}{R}\left(\frac{1}{2} + \frac{3}{4}\frac{X}{D} - \frac{1}{4}\frac{X^3}{D^3}\right)$$

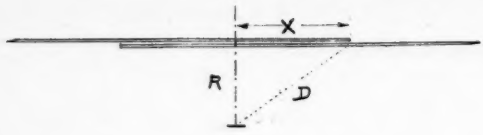


Fig. 11.

Illumination Below Centre of Finite Source

($\alpha = 0$). (Table II, Column 2) (See Fig. 6) (see also [3]).

This can be obtained from the previous expressions by superimposing two semi-infinite sources (and a negative infinite source) as shown in Fig. 11:

Illumination below centre: $E_c = 2\,E_{si} - J/R$, where E_{si} is the illumination below a semi-infinite source, namely,

$$E_{si} = \frac{J}{R}\,(1 - \triangle).$$ (For definition of $\triangle$ see Sec. 4.3.)

NOTE.—X is always the horizontal distance from the end of the source, i.e., half a lamp length of the finite source.

Illumination on Surface at Right Angles to a Finite Source

($\alpha = 90°$). (Table II, Column 3) (See Fig. 8).

In equation 3.1, $\cos(\alpha - \psi)$ becomes $\sin\psi$ for $\alpha = 90°$:

$$\textit{Case (i)}\quad E = \frac{I_\circ}{R}\int_{\psi=\cos^{-1}(R/D)}^{\psi=0} \sin\psi\,d\psi = \frac{I_\circ}{R}\Big|-\cos\psi\Big|_{\cos^{-1}(R/D)}^{0} = \frac{I_\circ}{R}\left(1-\frac{R}{D}\right)$$

$$= \frac{J}{2R}\left(1-\frac{R}{D}\right)$$

$$\textit{Case (ii)}\quad E = \frac{I_\circ}{R}\int_{\psi=\sin^{-1}(X/D)}^{\psi=0} \sin\psi\cos\psi\,d\psi = \frac{I_\circ}{R}\Big|\tfrac{1}{2}\sin^2\psi\Big|_{\sin^{-1}(X/D)}^{0}$$

$$= \frac{I_\circ}{R}\,\frac{1}{2}\,\frac{X^2}{D^2} = \frac{J}{\pi R}\,\frac{X^2}{D^2}$$

$$\textit{Case (iii)}\quad E = \frac{I_\circ}{R}\int_{\psi=\cos^{-1}(R/D)}^{\psi=0} \sin\psi\,\cos^2\psi\,d\psi = \frac{I_\circ}{R}\int_{m=R/D}^{m=1} -m^2\,dm,\ \textit{where}\ m = \cos\psi,$$

$$= \frac{I_\circ}{3R}\Big|-m^3\Big|_{R/D}^{1} = \frac{I_\circ}{3R}\left(1-\frac{R^3}{D^3}\right) = \frac{J}{4R}\left(1-\frac{R^3}{D^3}\right)$$

Appendix 5

Effect of Periodic Variation

Since most long linear sources are made up of single lamps or lamp elements of finite length, there is usually a periodic variation of light emission along the length of the linear source.

This in turn will result in a periodic fluctuation of the illumination on nearby objects. If for the calculation of illumination values the source is assumed to be

continuous, i.e., if this periodicity is neglected, a certain error is made and we may ask under what conditions this error is negligible.

This error is bound to decrease with the distance of the object surface from the source. It also depends on the degree of discontinuity and type of distribution of light from the source elements.

In order to get a quantitative idea of this error let us consider two specific cases:

(i) a "linear source" made up of flat diffusing point sources separated by entirely non-luminous gaps (e.g., consider a long passage lit by small plane diffusing ceiling panels);

(ii) a "linear source" made up of uniform point sources (e.g., a passage lit by spherical diffusing fittings).

The following table is obtained by direct calculation of the maximum and minimum illumination, E_{max} and E_{min}, on a plane H feet below the sources, which are spaced C feet apart.

E_{max} would occur straight below a source, E_{min} midway between sources.

The percentage fluctuation is defined as $100\ (E_{max} - E_{min})/E_{max}$.

Percentage Fluctuation of Illumination on a Parallel Plane below a Row of Point Sources

Source	Spacing/Height Ratio (C/H)			
	.7	1	1.4	2
Flat diffusing sources	.5%	5%	22%	52%
Spherical diffusing sources	.3%	3%	12%	35%

These two cases are rather extreme. The fluctuations in the case of a row of fluorescent lamps without louvres should be much less even if there are small gaps between the lamps. Only in the case of louvred lamps spaced at appreciable distances or in the case of shaded point sources the fluctuations would be larger.

On the other hand, louvring of lamps mounted end-to-end would not lead to serious fluctuation, since the distance between louvres is relatively small.

For most practical purposes we may conclude that the effect of periodic variations is negligible if $H > C$, i.e., if the object distance from the source exceeds the pitch of periodicity.

NOTE.—If periodic variations are present, Sector Flux is defined as *average* flux per unit length per radian. This is implied in equation (3.3).

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The Revealing Power of Street Lighting Installations and its Calculation*

By A. J. HARRIS, B.Sc., and A. W. CHRISTIE, M.A.

Summary

This paper extends the work of Waldram and others on the revealing power of street lighting installations. It is shown that values of revealing power, approximating closely to the true values, can be calculated from the mean pedestrian illumination E_m and the road surface luminance B. Curves are given showing this approximate revealing power as a function of the two quantities B and B/E_m. The curves may be used to find the revealing power to any desired accuracy for any type of installation by a simple calculation of averages. The broad effects of changes in the lighting installation may also be understood with the aid of these curves.

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ACKNOWLEDGEMENTS.
REFERENCES.

(1) Introduction

The conception of "revealing power" was introduced into the theory of street lighting by Waldram[1] to clarify ideas about the efficiency of installations. The lighted street is regarded as a means of revealing to the driver of a vehicle the objects which may be upon it. From this point of view the quality of the lighting will be judged by whether or not such objects are adequately revealed, rather than by the illumination or the luminance of the road surface.

Waldram, giving this idea a clear-cut meaning and defining the rules of calculation, called the percentage of objects revealed by any element of the road surface the revealing power of that element. For any street this could be exhibited by means of photographs showing the road surface mapped out into regions of different revealing power—high values in the bright regions, low values in the dark. Although it was not difficult to find the values for any given street, the revealing power appeared to depend so much on the variation in the lighting along the street that it was difficult to see its relationship to the other factors involved. It is shown, however, in Appendix I of this paper that, provided the span does not exceed about six times the mounting height (Group A lighting), the revealing power of an element of the background is in fact determined to within a few per cent. by two quantities only, the luminance B of the background, and E_m, the mean illumination of pedestrians in the street. As will be seen later, it is more convenient to use B/E_m as the second variable rather than E_m itself.

This paper develops some mathematical consequences of the ideas given in Waldram's original paper and does not in any way modify them. For details and a full discussion of the questions involved the original should, therefore, be consulted. The only change is that objects lighter than their background and objects darker than their background have been included in the total revealing power. Waldram

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omitted the lighter objects because their contribution was small. Revealing power calculated only from objects darker than the background is here called revealing power by silhouette and denoted by R. From objects brighter than the background is calculated revealing power by reversed silhouette, which is denoted by R′. The sum of these, R + R′, is called the total revealing power.

(2) Evaluation of Revealing Power

Objects in lighted streets usually appear darker than the road behind them, that is, they are seen in silhouette. Sometimes, however, they are brighter than the background and are then said to be seen in reversed silhouette. For an object to be seen adequately under any given set of conditions a certain difference in luminance between object and background is necessary. This is called the critical contrast, and if the actual contrast is less than this the object will not be adequately visible. The magnitude of the critical contrast will depend on the nature of the task: a driver may be dissatisfied with a degree of contrast which as a pedestrian he might regard as adequate for safety. The objects found in streets are of different shapes and sizes, but the contrast used by Waldram in calculating revealing power was obtained for a vertical disc of diameter 1.5 feet. By means of experiments on roads of almost uniform luminance Dunbar[2] determined the critical contrast appropriate to the motorist, that is, the contrast which a motorist driving at 30 m.p.h. and at a distance of more than 100 feet from the disc judged to be adequate for safety. His curve is reproduced in Fig. 1 where the critical contrast is shown as a fraction of the background luminance. Dunbar also showed that this critical contrast applied to objects of shapes other than circles, provided their smaller dimension was 1.5 feet.

In order to evaluate a revealing power consider a driver looking at a part of the road surface whose luminance is B foot lamberts. If a disc of luminance factor β is placed at some point in his line of sight it will receive some illumination E and appear of luminance $E\beta$. If the contrast between the two luminances B and $E\beta$ exceeds the critical contrast determined by Dunbar then the disc is adequately visible, otherwise it is not. It is next assumed that the disc has an equal chance of being anywhere along the line of sight, and that the probability of its having the luminance

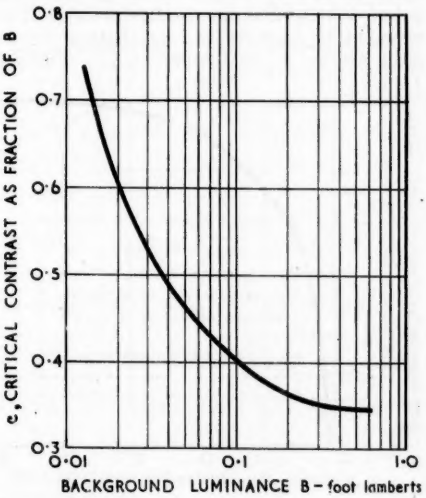

Fig. 1. Standard curve for critical contrast. (Dunbar).

factor β is the same as that for pedestrian clothing. The distribution of luminance factors for pedestrian clothing has been determined by Smith[3] and is reproduced in Fig. 2. It is then possible, if the variation of E along the line of sight is known, to evaluate (as in Appendix I) the chance that a disc chosen at random in this way would be adequately visible or, in other words, the proportion of such discs which would be revealed. The proportion expressed as a percentage is by definition the revealing power. In calculating the revealing power by reversed silhouette it is assumed that the critical contrast is the same for objects brighter than the background as it is for objects darker than the background, for which alone Dunbar's values were determined.

To simplify the calculation the variation of E across the road is neglected since it is usually unimportant. The variation along the line of sight is taken into account by Waldram because it is usually large, although it is assumed for simplicity to be the same as the variation along a single span of the installation whatever the length of the line of sight. However, it is shown in Appendix I that this variation can also be largely neglected, for Group A lighting. Those points where the illumination is high are largely compensated for by those where it is low, the resultant revealing power depending as far as E is concerned almost entirely on the mean value E_m. The results obtained in Appendix I when the variation of E is neglected will now be discussed.

(3) Revealing Power as a function of B and B/E_m

Fig. 3 gives approximately the revealing power of any installation. It is calculated from Equations (4) (5) and (6) of Appendix I. The full curves are for the total revealing power R + R′ and the broken curves for R, the revealing power by silhouette. For an installation in which pedestrian illumination is uniform the curves are exact; for other installations they are approximate. In existing streets the variation in pedestrian illumination is large and a discussion of the errors in the curves due to this non-uniformity may be found in Appendix I. It is there shown that for any likely system of Group A lighting, in which the span will not exceed six times the mounting height of the lamps, the error is not likely to exceed 2.5 per cent. Some idea of the magnitude of the error may be obtained from Fig. 4 calculated for a low background luminance—since it is at low luminances that the error is most pronounced—and for two installations representing fairly extreme types beween which most Group A installations fall. The errors for Group A lighting are therefore

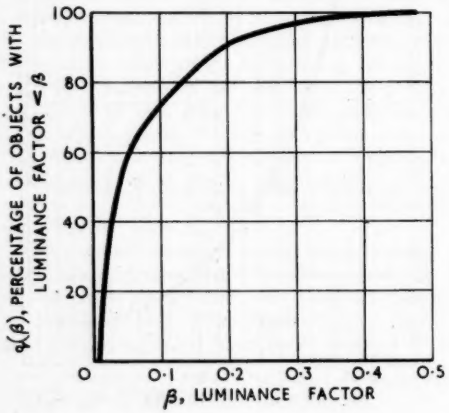

Fig. 2. Standard curve for luminance factors of clothing. (Smith).

unlikely to exceed the largest values given in Fig. 4, and, indeed, the distributions used in Group A lighting approximate fairly closely to the type which gives the smaller error, the secc. θ distribution. If the curves of Fig. 3 were corrected for the variation in E it seems likely that the correction would be very similar in nature for most common distributions, namely, a raising of the left-hand part of the curves and a smaller depression of the right-hand, the general character of the curves being unaltered.

Fig. 3, although approximate when applied directly to a whole street, may be used if desired to find the revealing power more accurately by sub-dividing the span into a number of sections, obtaining the individual means of E for the sections, and averaging the corresponding revealing powers obtained from Fig. 3. The formula is given in Appendix II; it is probably the quickest way of getting accurate values of revealing power and may be applied to any installation or length of span.

By plotting the revealing power against B/E_m instead of against E_m itself, all the curves have been made to run more or less parallel, and it is evident that because the critical contrast itself is changing only very slowly the curves for the higher values of B tend to lie very close together. For B greater than about 0.1 foot lambert, the revealing power, as was pointed out by Waldram and Dunbar, does not vary by more than a few per cent., so that to a very considerable degree of accuracy the revealing power is determined by a single variable B/E_m. Thus for streets whose luminance patterns are fairly similar, and in which the luminance does not fall much below 0.1 foot lambert, a quantity of fundamental importance to the revealing power is B_m/E_m, where B_m is some representative luminance, for example, the maximum or the mean. Except for very low values, the greater the quantity B_m/E_m the higher is the revealing power. This should apply particularly to streets whose surfaces are fairly uniform in brightness.

Fig. 3 gives a fairly close picture of the revealing power in any system of Group A lighting. Of course, the revealing powers encountered in the street will depend on what luminances the installations actually produce, but for a given luminance B the curves of Fig. 3 will give the corresponding revealing power. In general the curves slope upward towards the right, that is, the total revealing power and the revealing power by silhouette increase as B/E_m increases, while the contribution of reversed

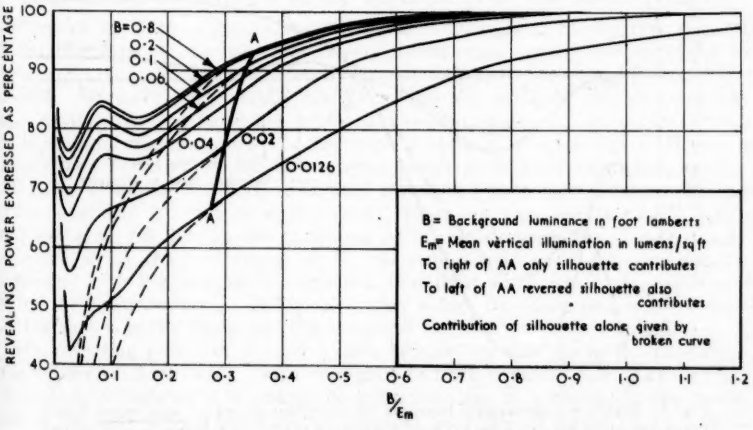

Fig. 3. Revealing power as a function of B/E_m for constant values of B. Values of B marked alongside curves.

silhouette diminishes until to the right of the line AA, that is for B/E_m greater than about 0.3, it vanishes. The exact point at which the contribution of reversed silhouette vanishes is shifted to the right by the variation of E about its mean, but for B/E_m greater than 0.3 revealing power by reversed silhouette does not account for more than about 2 per cent.

Seeing by reversed silhouette is not generally considered to be important in Group A installations; that is, in Fig. 3 values of B/E_m less than 0.2 are seldom encountered. While this may be true when the background under consideration is a dry road surface, it may not be true for other backgrounds such as a wet road surface, buildings or hedges alongside the road. Where revealing power by silhouette falls to low values reversed silhouette steps in to prevent a catastrophic drop in the total revealing power. For this reason changes of fashion, which affect the distribution of luminance factors of clothing and therefore the revealing power, are likely to have much less effect on the total revealing power than on the revealing power by silhouette alone.

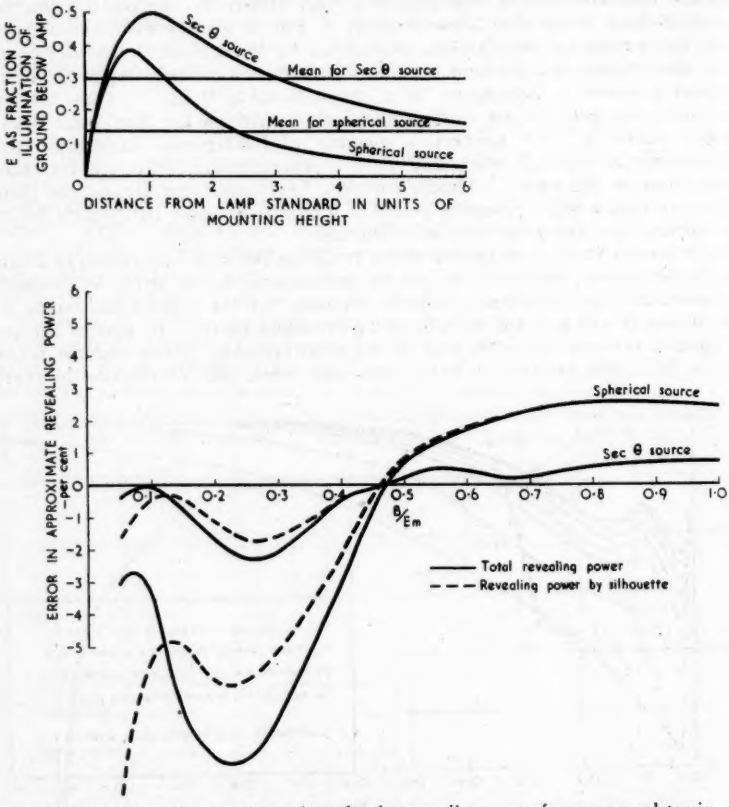

Fig. 4. Errors in approximate formulae for revealing power for span equal to six times mounting height, a background luminance of 0.02 foot-lamberts and for two distributions, (i) a spherical distribution, and (ii) a distribution with candlepower proportional to sec θ.

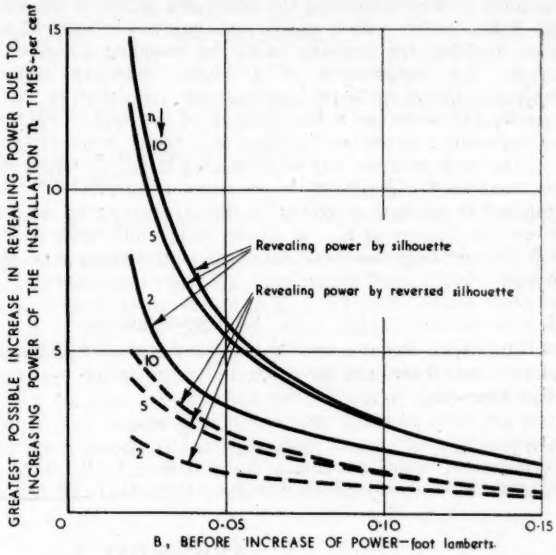

Fig. 5. Effect on revealing power of increasing power of installation *n* times.

At the extreme left of the diagram the curves rise again sharply to 100 per cent., but here the objects are seen entirely by reversed silhouette. This end of the diagram corresponds to lighting of the kind obtained from motor vehicle headlights rather than to any normal type of street lighting. The oscillations near B/E_m equal to 0.1 in the curves of total revealing power are due to two rather sudden changes of slope in the curve of the luminance factor, Fig. 2.

(4) Effects of Changes in the Installation or the Road Surface

We may see from these curves the effect of changes in the lighting installation. Consider, for example, the effect of raising the intensity without changing the distribution. If the intensity is increased n times then B becomes nB but B/E_m is unaltered. We have already seen that at the highest values of B this constancy of B/E_m means that the revealing power is almost unchanged. In general, however, there is a slight increase. This is discussed at the end of Appendix I. Fig. 5 gives an upper limit for the increase in revealing power made possible in this way. For $B = 0.1$, for example, the increase in total revealing power cannot exceed 4 per cent. even for a tenfold increase of illumination. Doubling the intensity cannot add more than 5 per cent. for luminances exceeding 0.05. In deriving Fig. 5 no assumptions were made as to the uniformity of E; the values hold good for any installation. Comparison with Fig. 3 suggests, however, that these limits are likely to be reached only for the lower values of B/E_m; at higher values the increase is a good deal less than that shown in the figure and this is indeed true whatever the installation.

The revealing power of a road is also affected by changes in the reflecting properties of the road surface. Such changes affect the value of B/E_m as well as that of B and, in consequence, a change of luminance due to a change in the luminance factor of the road surface has a greater effect on the revealing power of the installation than the same change of luminance produced by an alteration in the intensity of the light Consider, for example, the point $B = 0.1$, $B/E_m = 0.2$ in Fig. 3. Doubling

the value of B by increasing the luminance factor of the road surface changes B to 0.2 and B/E_m to 0.4, with a rise in revealing power from 82 to 93.5 per cent. Doubling B by doubling the intensity raises the revealing power only to 83.5 per cent. Conversely, the replacement of a highly reflecting surface by one with a low luminance factor makes a much greater reduction in revealing power than does a considerable reduction in the intensity of the light. Such considerations demonstrate the importance of the road surface as a factor in the proper illumination of the street.

The most effective way of increasing B/E_m, by reducing E_m without reducing B, has been tried experimentally on one-way streets. This system of unidirectional lighting has no light projected in the same direction as the traffic, so that as far as drivers are concerned E_m is almost zero. Silhouette seeing is very good, the values of B/E_m are large, and revealing powers of 100 per cent. should be attainable without difficulty.

Acknowledgements

The paper was prepared at the Road Research Laboratory of the Department of Scientific and Industrial Research and is published by permission of the Director of Road Research.

References

(1) Waldram, J. M., Trans. Illum. Eng. Soc. (London), **3**, 173 (1938).
(2) Dunbar, C., Trans. Illum. Eng. Soc. (London), **3**, 187 (1938).
(3) Smith, F. C., Trans. Illum. Eng. Soc. (London), **3**, 196 (1938).

APPENDIX I

Approximate Calculation of Revealing Power

It will be shown that for Group A lighting, in which the span does not exceed about six times the mounting height, the revealing power may be calculated to within a few per cent. from an expression which neglects the variation in E along the street, where E is the pedestrian illumination, that is, the illumination of vertical surfaces in the street.

Fig. 1 gives the critical contrast c as a fraction of the background luminance B. Objects darker than b or brighter than b′ are adequately visible against a background of luminance B where

$$b = (1 - c)B$$
$$b' = (1 + c)B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Objects having a luminance factor less than b/E have a luminance less than b when the illumination is E. Similarly, under illumination E, objects with luminance factors greater than b'/E have luminance greater than b′. These objects are therefore adequately visible under illumination E.

For pedestrian clothing the distribution of β, the luminance factor, is given in Fig. 2. Assuming that the objects under consideration have the same distribution, the percentage adequately visible under illumination E and against a background of luminance B is $q(b/E)$ for vision by silhouette and $100 - q(b'/E)$ for vision by reversed silhouette.

If the pedestrian illumination E is distributed along a span of the installation so that a fraction $p(E)dE$ of the span has illumination between E and $E+dE$ then, of objects in the line of sight, the percentage visible in silhouette is

$$R = \int_{E_1}^{E_2} q(b/E)\ p(E)dE \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

and in reversed silhouette

$$R' = 100 - \int_{E_1}^{E_2} q(b'/E)\ p(E)dE \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where

R is the revealing power as defined by Waldram and here called revealing power by silhouette,

R′ is the revealing power by reversed silhouette, and

R + R′ is the total revealing power.

Expansion of q(b/E) in (2) as a power series in $\frac{E - E_m}{b}$, where E_m is the mean value of E in the span, and subsequent integration leads to an infinite series for R in which the first term depends on the mean value E_m, the second term vanishes and the third depends on the variation of E about its mean E_m. Unfortunately these two non-vanishing terms, although adequate to deal with quite large variations in E, are not enough to give an accurate value of the revealing power under all conditions when applied to such variations as are common in present-day streets, and indeed the first term alone is sometimes more accurate than the first two. There is also, as described in Appendix II, a more convenient way of getting accurate values of revealing power than by using the series, so we shall consider only the first term. This approximation is obtained merely by substituting E_m for E in q(b/E) in the integrals. The expression $q(b/E_m)$ is then a constant and may be brought outside the integral sign. Thus we get:

$$R \simeq q(b/E_m) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$R' \simeq 100 - q(b'/E_m) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$R + R' \simeq 100 + q(b/E_m) - q(b'/E_m) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

To obtain an idea of the errors involved in these approximations, they have been applied to two types of beam distribution which represent fairly extreme forms. These are a spherically symmetrical distribution giving a constant intensity in all directions, and one giving an intensity proportional to sec θ where θ is the angle between the beam and the downward vertical. The illumination of vertical surfaces at ground level due only to the nearest lamp in these installations was obtained for spans of six and twelve times the mounting height. In the shorter span, as may be seen in Fig. 4, the maximum value of E is about 16 times the minimum in the spherical distribution. In the other distribution it is only about three times. Using the method of Appendix II to calculate accurate values of revealing power the errors in the values given by (4) and (6) were obtained. The results for a span equal to six times the mounting height, and a background luminance of 0.02 foot lambert, are shown in Fig. 4. The spherical distribution gives the largest error. Except possibly for very low values of B/E_m it does not exceed about 10 per cent. For higher background luminances and more uniform pedestrian illuminations the errors are smaller. Even for spans of 12 times the mounting height the error for B/E_m greater than 0.4 hardly exceeds 4 per cent., though at the lower values it rises to almost 20 per cent. for the spherical distribution.

The distributions used in Group A lighting tend to the sec θ type and direct a larger proportion of light along the road than does the spherical distribution. In consequence the pedestrian illumination is more uniform than under the worst conditions considered here. Thus, for Group A lighting, in which the span does not normally exceed six times the mounting height, the approximate formulae should give the revealing powers with errors which are less than about 2.5 per cent. for B/E_m greater than 0.4 and may rise to about 10 per cent. at lower values. It is very probable that the errors will not exceed 2.5 per cent. for any value of B/E_m.

Since the critical contrast given in Fig. 1 tends to a constant as the background luminance increases, the revealing powers in an installation also tend to constant values

if the output of the installation is increased. An upper limit to the increase in revealing power which may be obtainable by increasing the output n times is easily obtained. This is shown in Fig. 5; it appears, from the particular installation represented by Fig. 3, that the limit is not necessarily attained.

APPENDIX II

A Simple Method of Obtaining Accurate Values of Revealing Power from the Approximate Curves of Fig. 3

If for some purpose it is desired to find revealing power more accurately than it is given in Fig. 3, or if the installation is not of Group A type and has a span greatly exceeding six times the mounting height, an accurate value may be found by subdividing the span. Suppose the length L of the span is divided into lengths l_1, l_2 . . . . and the mean values of E within these sections are E_1, E_2 . . . . then the total revealing power R + R′ is given by

$$R + R' = \frac{(R_1 + R'_1)\, l_1 + (R_2 + R'_2)\, l_2 + \ldots .}{L}$$

where $(R_1 + R'_1)$, $(R_2 + R'_2)$ . . . are the total revealing powers read off from Fig. 3 corresponding to the values B/E_1, B/E_2. . . . The formulae for R or R′ separately are similar.

For a span equal to six times the mounting height, a subdivision into two equal lengths gives the revealing power for a spherical distribution to within about 2 per cent. over the range $B/E_m > 0.1$. It is unlikely that subdivision need be carried much beyond this. To judge the accuracy of any such calculation a further subdivision may be made. If the new value does not differ significantly from the old, the process of subdivision has been carried far enough.

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Temple-avenue and
Tudor - street,
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